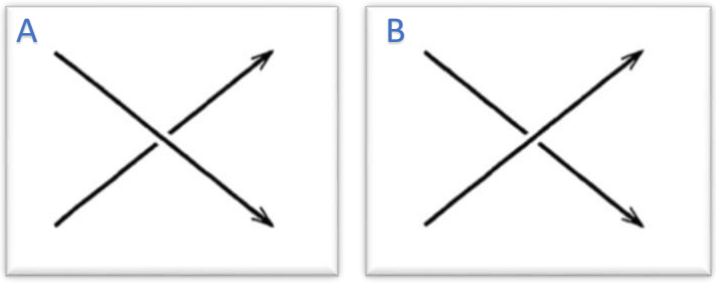
Chapter 1 Notes

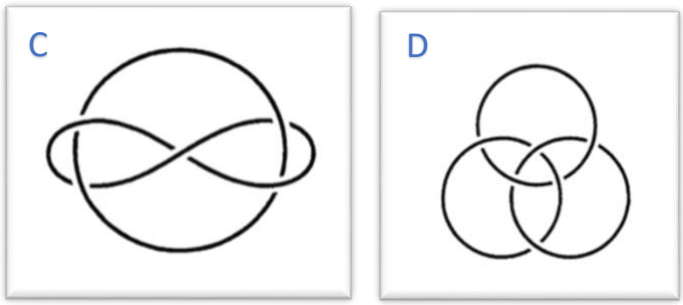
1. Key Concepts

Term	Definition	Source
Knot	A closed curve in space that does not intersect itself anywhere.	Page 2
Unknot	Also known as the trivial knot, simply an unknotted circle.	Page 2
Projection	Picture of a knot from a certain direction.	Page 3
Crossings	The places where the knot crosses itself in projection.	Page 3
Topology	The study of the properties of geometric objects that are preserved under deformations.	Page 6
Alternating knot	A knot with a projection that has crossings that alternate between over and under as one travels around the knot in a fixed direction.	Page 7
Composition of Knots	A knot obtained by removing a small arc from each knot projection and then connecting the four endpoints by two new arcs. Composition is denoted by #.	Page 7
Composite Knot	A knot that can be expressed as the composition of two knots, neither of which is the trivial knot.	Page 8
Factor Knots	The knots that make up the composite knot.	Page 8
Prime Knot	A knot that is not the composition of any two nontrivial knots.	Page 9
Orientation	A direction to travel around the knot.	Page 10
Invertible Knot	A knot that can be deformed back to itself so that an orientation on it is sent to the opposite orientation.	Page 11
Ambient Isotopy	The movement of the string through three-dimensional space without letting it pass through itself.	Page 12
Planar Isotopy	A deformation of a knot projection within the projection plane.	Page 12
Reidemeister Move	Three movements to change a projection of the knot that will change the relation between the crossings.	Page 13
Reidemeister Move Type I	Putting in or taking out a twist in the knot.	Page 13
Reidemeister Move Type II	Adding two crossings or removing two crossings by moving a strand under or over another strand.	Page 13
Reidemeister Move Type III	Sliding a strand of the knot from one side of a crossing to the other side of the crossing.	Page 13

Term	Definition	Source
Amphicheiral	A knot that is equivalent to its mirror image.	Page 15
Link	A set of knotted loops all tangled up together.	Page 17
Link of Two Components	A link made up of two loops knotted with each other. I.e.: Whitehead link.	Page 17
Splittable Link	A link made up of components that can be deformed so that they lie on different sides of a plane in three-space.	Page 17
Linking Number	Numerical measurement of how linked up two components are calculated by dividing the sum of all crossings by 2.	Page 18-19
Invariant	A property of the link that is unchanged by ambient isotopy once the orientations are chosen on the components of the link.	Page 21
Brunnian Link	A link that is nontrivial itself, but the removal of any one of the components results in a set of trivial unlinked circles.	Page 22
Tricolorability	Each of the strands in the projection can be colored one of three different colors, so that at each crossing, either three different colors come together, or all the same color comes together.	Page 23

2. Key Figures





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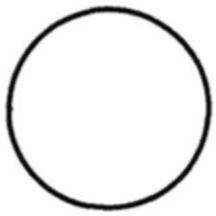

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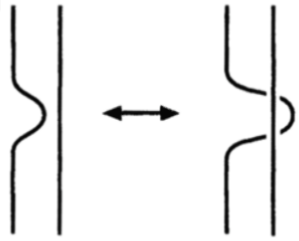

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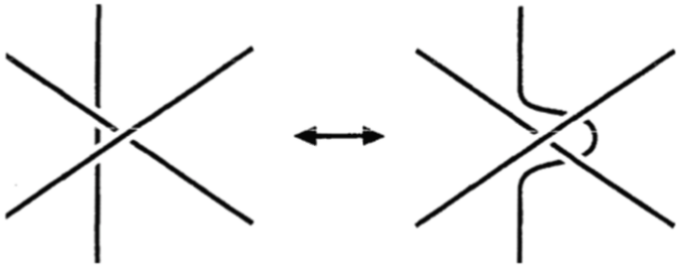

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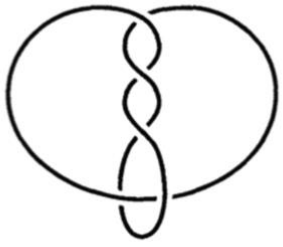
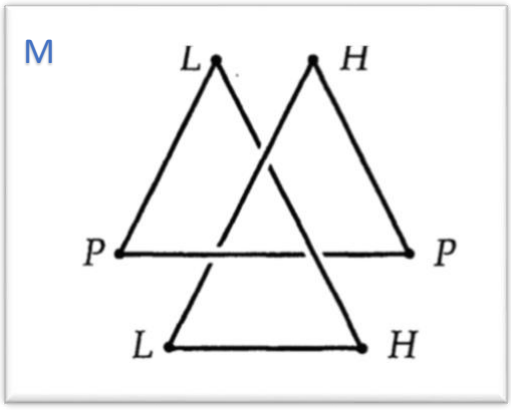


K



Matching: What are these figures?

( ) 1. Reidemeister Move Type I

( ) 2. Trefoil knot made from six sticks

( ) 3. Composite knot made up of two trefoil knots

( ) 4. Reidemeister Move Type III

( ) 5. Brunnian link of four components

( ) 6. Figure-eight knot

( ) 7. Reidemeister Move Type II

( ) 8. Borromean rings

( ) 9. Unkot

( ) 10. +1 crossing (right hand)

( ) 11. Whitehead link

( ) 12. -1 crossing (left hand)

( ) 13. Hopf link

Answers:

1. G 2. M 3. K 4. I 5. L 6. J 7. H 8. D 9. E 10. A 11. C 12. B 13. F