Advancing mathematics by guiding human intuition with AI

- Objective of the framework: identify a function which associate X(z) to Y(z)
- 2 stages
  - Supervised learning: train a function f that predicts Y(z) using only X(z)
  - Attribution techniques: using gradient saliency to calculate the derivative of f(X(z)) with respect to X(z)

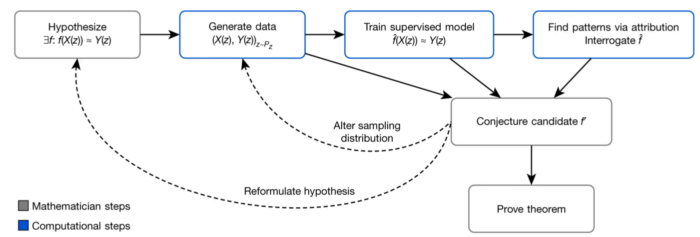

Fig 1, Davies *et al*.

Supervised learning:
- Training data is labelled and we seek to predict outputs given inputs
- Classification - where outputs are discrete
- Regression - where the outputs are real-valued
- Contribution of this stage: learning non-linear functions
- If $\hat{f}$ is more accurate than would be expected by chance, there exists such a relationship between X(z) and Y(z)

Attribution techniques:
- Quantify component of X(z) that $\hat{f}$ is sensitive to
- Calculate how much $\hat{f}$ changes in predictions of Y(z) given small changes in X(z)

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- Application in Topology

z: **Knot**	$X(z)$: **Geometric invariants**				$Y(z)$: **Algebraic invariants**		
	Volume	Chern–Simons	Meridional translation	...	Signature	Jones polynomial	...
	2.0299	0	i	...	0	$t^{-2} - t^{-1} + 1 - t + t^2$	...
	2.8281	−0.1532	$0.7381 + 0.8831i$	...	−2	$t - t^2 + 2t^3 - t^4 + t^5 - t^6$	...
	3.1640	0.1560	$-0.7237 + 1.0160i$	...	0	$t^{-2} - t^{-1} + 2 - 2t + t^2 - t^3 + t^4$	...

We hypothesized that there was a previously undiscovered relationship between the geometric and algebraic invariants.

Fig 2, Davies *et al*.

Invariants: geometric or numerical quantities that are the same for two equivalent knots.

Notable example conjecture: the hyperbolic volume of a knot (geometric invariant) should be encoded within the asymptotic behaviour of its coloured Jones polynomials (algebraic invariants).

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- Application in Topology
  - Prediction target: σ(K)
  - Three most relevant invariants: Re(μ), Im(μ), λ
  - Natural slope: slope(K) = Re(λ/μ) linearly related to σ(K)
  - Conjectures:

$$|2\sigma(K) - \mathrm{slope}(K)| < c_1 \mathrm{vol}(K) + c_2 \qquad |2\sigma(K) - \mathrm{slope}(K)| \le c\,\mathrm{vol}(K)\mathrm{inj}(K)^{-3}$$

  - Insights: the signature controls the non-hyperbolic Dehn surgeries on the knot and that the natural slope controls the genus of surfaces in ℝ4+ whose boundary is the knot.

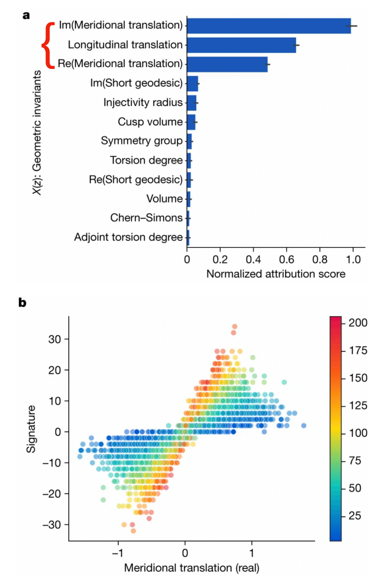

Fig 3, Davies *et al*.

meridional translation μ and the longitudinal translation λ

Dehn surgeries : diff ways of filling in a loop of the meridian

Inj: injectivity radius

Complement 3D space - the knot