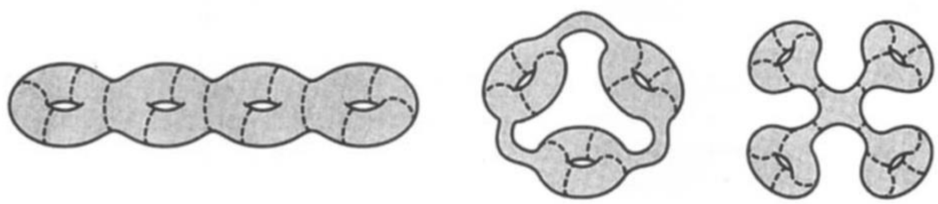
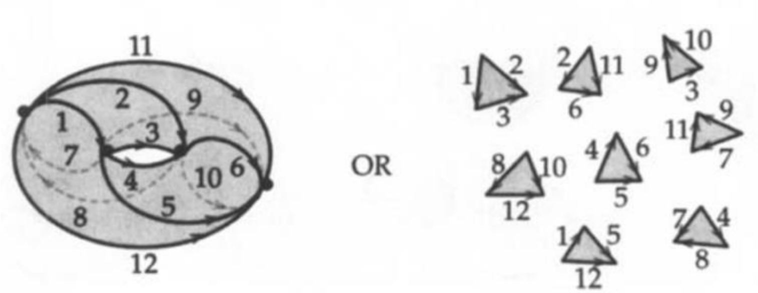
Chapter 4 Notes

1. Surfaces without Boundary

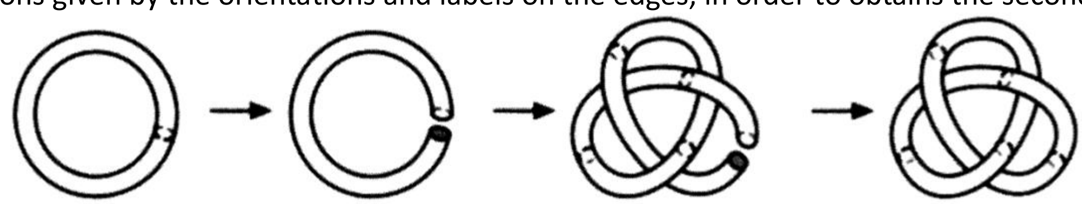
Two-manifold: aka surface, is defined to be any object such that every point in that object has a neighborhood in the object that is a disk.
Isotopy: continuous deformation of a knot or surface without cutting or breaking such that the initial and final knots (or surfaces) are equivalent. Some examples are rubber deformations like stretching, twisting, bending, or shrinking.
Isotopic surfaces: two equivalent surfaces in space under isotopy.



Triangulation: division of a surface into triangles.

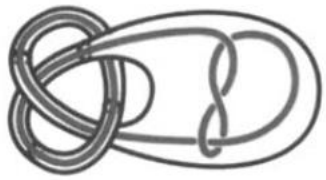


Homeomorphic: for two surfaces, if one of them can be triangulated, then cut along a subset of the edges into pieces, and then glued back together along the edges according to the instructions given by the orientations and labels on the edges, in order to obtains the second



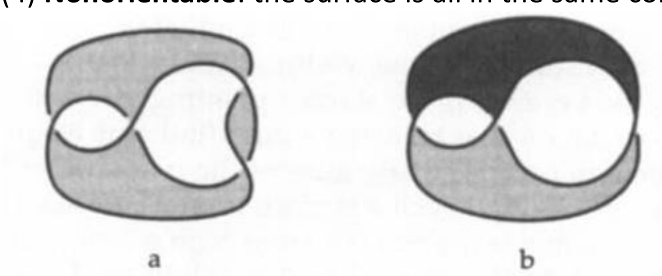
surface.
Genus: the number of holes in the doughnut. A sphere has genus 0. A torus has genus 1.
Embedding: the choice of how to place a surface in space.
Euler characteristic: X = V – E + F. V the number of vertices in the triangulation of the surface. E the number of edges. F the number if triangles.
Connected sum: an operation to obtain a new surface from multiple knots by removing a small segment of each knot then gluing the loose ends together.
(10) **Compact surface**: has a triangulation with a finite number of triangles.
(11) **Complement of a knot**: the leftover if we drill the knot out of space.
(12) **Compression**: an operation that obtains the tightest enclosing surface.

(13) **Swallow-follow torus**: an incompressible torus of a composite knot that swallows one of the two factor knots and follows the other one.



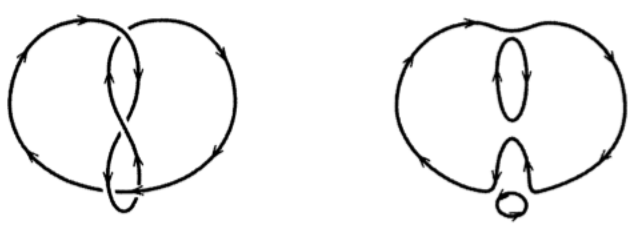
2. Surfaces with Boundary

(1) **Boundary components**: circular boundaries left in the surfaces.
(2) **Capping off**: filling in boundary components with disks.
(3) **Orientable**: if a surface in 3D space has two sides that can be painted different colors so that the colors never meet except along the boundary of the surface. i.e.: a torus.
(4) **Nonorientable**: the surface is all in the same color. i.e.: a Möbius band.



3. Genus and Seifert Surfaces

(1) **Seifert circles**: circles resulted from eliminating all crossings in a knot.



(2) **Seifert surface for a knot**: an orientable surface with the knot being the one boundary component of the surface.
(3) **Theorem of composition**: g(J#K) = g(J) + g(K) where J and K are two knots and g is the genus.